

Excercises 7

Excercise 1: Truth tables

- Construct truth tables for the following statements. Note whether any are logically equivalent.

- (1)
- $(p \vee (\neg q))$
 - $(\neg((\neg p) \wedge q))$
 - $((p \leftrightarrow q) \wedge p)$
 - $((p \rightarrow (q \vee (\neg r))) \wedge (p \rightarrow (q \vee (\neg r))))$
 - $((p \rightarrow q) \rightarrow p) \rightarrow q$

Excercise 2: Tautology, contradiction, contingency

- Let p , q , and r be atomic statements. Which of the following are tautologies, contradictions, or contingent statements?

- (2)
- $(p \vee (\neg p))$
 - $(p \vee q)$
 - $((p \wedge q) \rightarrow (p \vee r))$
 - $((\neg p) \wedge (\neg(p \rightarrow q)))$
 - $((p \vee r) \rightarrow (\neg p))$

Excercise 3: Definition of connectives

- Certain of the logical connectives can be defined in terms of others. Example: $(p \rightarrow q)$ can be defined as $((\neg p) \vee q)$ (i.e. \rightarrow is expressible in terms of \neg and \vee), since the two statements are logically equivalent.
- Define \rightarrow in terms of \wedge and \neg .
- Define \wedge in terms of \vee and \neg .
- Define \leftrightarrow in terms of \rightarrow and \wedge .
- Show how the five connectives can be reduced to \wedge and \neg .

Excercise 4: Laws of statement logic

- Prove the following equivalence: $((p \wedge q) \vee p) \Leftrightarrow p$.
- Use the laws of statement logic (and, possibly, the equivalence you proved previously) to reduce each of the following statements to the simplest equivalent statement.

- (3)
- $((\neg p) \vee (p \wedge q))$
 - $((\neg p) \wedge q) \vee (\neg(p \vee q))$
 - $((\neg p) \wedge ((p \wedge q) \vee (p \wedge r)))$
 - $((\neg p) \wedge q) \leftrightarrow (p \vee q)$