

Excercises 2

Excercise 1:

- Besides the list notation and the predicate notation there is a third way to define sets: by recursive rules. An example for such a definition is given in (1-b), which defines the set (of all even natural numbers greater than or equal to 4) given by list notation in (1-a):

- (1) a. $E = \{4, 6, 8, 10, \dots\}$
b. (i) $4 \in E$,
(ii) if $x \in E$, then $x + 2 \in E$.

- (1-b) contains two clauses: the base clause (i), which defines one concrete element as being a member of E (namely 4), and the recursive clause (ii), which allows to generate all other members of E on the basis of 4.
- This is called a recursive definition because the clause (ii), which is supposed to define the members of E , makes reference to (one property of) E already. Note that the definition is not circular because of the existence of the base clause (i).
- Give two definitions for each of the following sets, one in terms of predication and one in terms of recursive rules.

- (2) a. $A = \{5, 10, 15, 20, \dots\}$
b. $B = \{7, 17, 27, 37, \dots\}$
c. $C = \{300, 301, 302, \dots, 399, 400\}$
d. $D = \{3, 4, 7, 8, 11, 12, 15, 16, 19, 20, \dots\}$
e. $E = \{0, 2, -2, 4, -4, 6, -6, \dots\}$
f. $F = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$

Excercise 2:

- Given the sets in (3), answer the questions in (4).

- (3) a. $S_1 = \{\emptyset, \{A\}, A\}$
b. $S_2 = A$
c. $S_3 = \{A\}$
d. $S_4 = \{\{A\}\}$
e. $S_5 = \{\{A\}, A\}$
f. $S_6 = \emptyset$
g. $S_7 = \{\emptyset\}$
h. $S_8 = \{\{\emptyset\}\}$
i. $S_9 = \{\emptyset, \{\emptyset\}\}$

- (4) Of the sets S_1 – S_9 , ...
a. ... which are members of S_1 ?
b. ... which are subsets of S_1 ?
c. ... which are members of S_9 ?
d. ... which are subsets of S_9 ?
e. ... which are members of S_4 ?
f. ... which are subsets of S_4 ?

... to be continued on next page \leftrightarrow

Excercise 3:

- Specify each of the sets in (5) by listing its members:

- (5)
- | | | | |
|----|--------------------|----|----------------------|
| a. | $\wp(\{a, b, c\})$ | d. | $\wp(\{\emptyset\})$ |
| b. | $\wp(\{a\})$ | e. | $\wp(\wp(\{a, b\}))$ |
| c. | $\wp(\emptyset)$ | | |

Excercise 4:

- Given the sets in (6), list the members of the sets in (7).

- (6)
- | | | | |
|----|----------------------------|----|------------------------------|
| a. | $A = \{a, b, c, 2, 3, 4\}$ | e. | $E = \{a, b, \{c\}\}$ |
| b. | $B = \{a, b\}$ | f. | $F = \{\}$ |
| c. | $C = \{c, 2\}$ | g. | $G = \{\{a, b\}, \{c, 2\}\}$ |
| d. | $D = \{b, c\}$ | | |
- (7)
- | | | | | | |
|----|------------|----|------------|----|---------|
| a. | $B \cup C$ | g. | $A \cap E$ | l. | $B - A$ |
| b. | $A \cup B$ | h. | $C \cap D$ | m. | $C - D$ |
| c. | $D \cup E$ | i. | $B \cap F$ | n. | $E - F$ |
| d. | $B \cup G$ | j. | $C \cap E$ | o. | $F - A$ |
| e. | $D \cup F$ | k. | $A - B$ | | |
| f. | $A \cap B$ | | | | |

Excercise 5:

- Given the sets in (6), and assuming that the universe of discourse is defined as $\bigcup\{A, B, C, D, E, F, G\}$, list the members of the following sets:

- (8)
- | | | | |
|----|---------------------------|----|---------------------|
| a. | $(A \cap B) \cup C$ | h. | $D' \cap E'$ |
| b. | $A \cap (B \cup C)$ | i. | $F \cap (A - B)$ |
| c. | $(B \cup C) - (C \cup D)$ | j. | $(A \cap B) \cup U$ |
| d. | $A \cap (C - D)$ | k. | $(C \cup D) \cap U$ |
| e. | $(A \cap C) - (A \cap D)$ | l. | $C \cap D'$ |
| f. | G' | m. | $G \cup F'$ |
| g. | $(D \cup E)'$ | | |