Excercises 11

Excercise 1: Bound vs. free variables
- For each of the expressions below, state whether the statement is open. Name the free variables (i.e. the variables that are not bound).

(1)   a. $(\forall x)(P(x) \vee Q(x, y))$
          b. $(\forall y)(Q(x) \rightarrow (\forall z)P(y, z))$
          c. $(\forall x)(P(x) \rightarrow (\exists y)(Q(y) \rightarrow (\forall z)R(y, z)))$

Excercise 2: Translation from English into predicate logic
- Translate the following English sentences into predicate logic. Choose your own variables and predicate letters, giving the key.

(2)   a.   Susan will go jogging only if Bill doesn't fall sick.
          b.   Leipzig is in Sachsen.
          c.   Jill likes red shoes.
          d.   Some girls like red shoes.
          e.   All people detest cold houses, especially when they are sick!
          f.   When John saw his friend who had tricked him, he got very angry.
          g.   No one saw any red boots.

Excercise 3: Well-formed formulas
- For the following expressions, say whether they represent a well-formed formula (wff) in predicate logic or not. Explain your answer.

(3)   a. $(\forall x)P(x)$
          b. $(\forall x)P$
          c. $(P(Q(x)) \rightarrow (\exists y)F(y, x))$
          d. $(x \wedge (y \vee z))$
          e. $(\forall x)(\exists x)(P(x, y) \vee Q(j, m))$
          f. $((\forall y) \rightarrow (\exists z))$
          g. $(\exists x)(P(x, z, j) \rightarrow (\forall y)(\neg K(f)))$
          h. $(P(y) \leftrightarrow (J(k) \wedge (Q(p) \vee (\neg Y(y)))))$
          i. $(\forall x)(\exists y)P(x \wedge y)$
          j. $(P \wedge Q(x))$
          k. $(\forall x)s(x)$
          l. $(\exists y)K(m)$